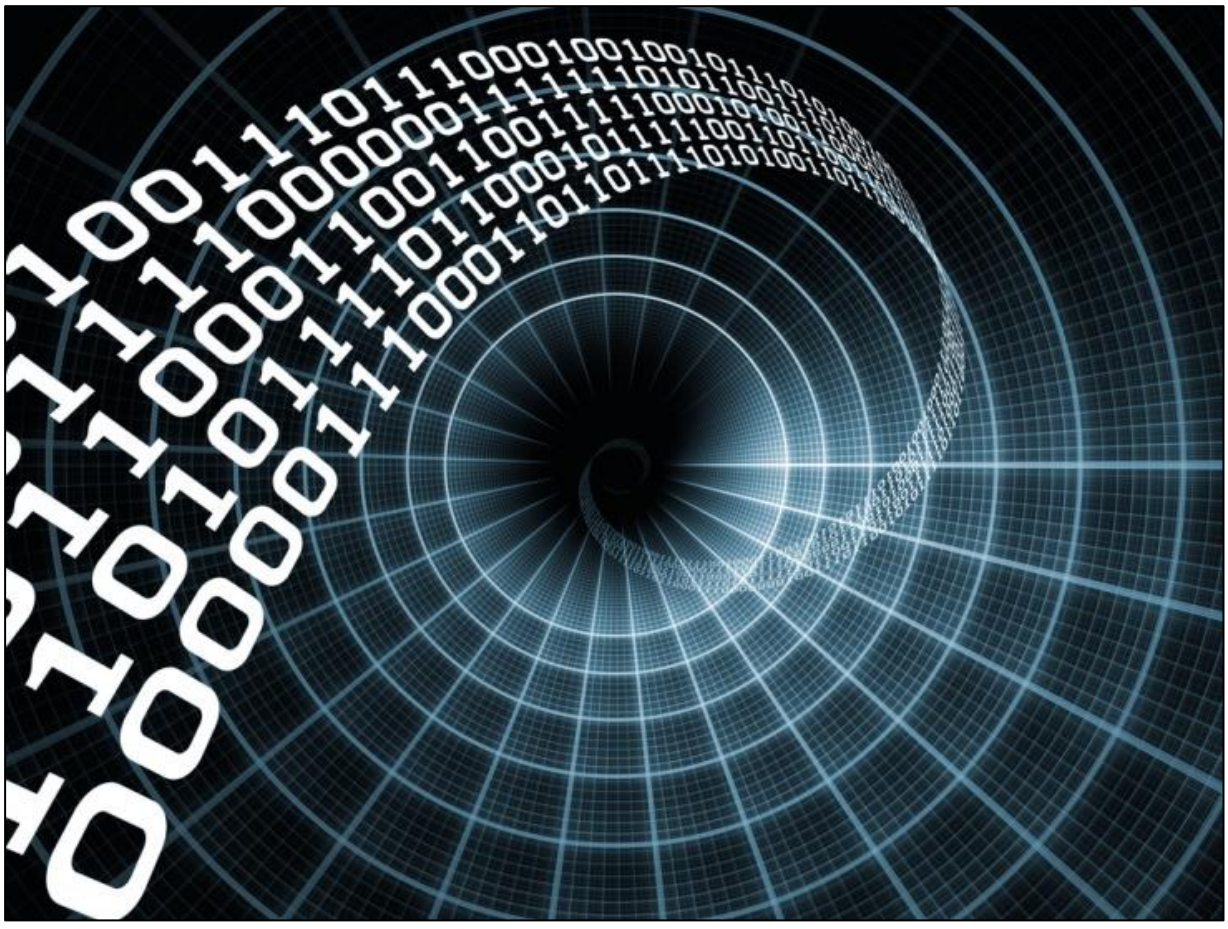
Course Support Notes

Higher Mathematics Course Support Notes



Please refer to the note of changes at the end of this document for details of changes from previous version (where applicable).

Contents

Introduction

These support notes are not mandatory. They provide advice and guidance on approaches to delivering and assessing the Higher Mathematics Course. They are intended for teachers and lecturers who are delivering the Course and its Units. They should be read in conjunction with the *Course Specification*, the *Course Assessment Specification* and the *Unit Specification*s for the Units in the Course.

General guidance on the Course

Aims

Mathematics is important in everyday life, allowing us to make sense of the world around us and to manage our lives.

Using mathematics enables us to model real-life situations and make connections and informed predictions. It equips us with the skills we need to interpret and analyse information, simplify and solve problems, assess risk and make informed decisions.

The Course aims to:

- motivate and challenge learners by enabling them to select and apply mathematical techniques in a variety of mathematical situations
- develop confidence in the subject and a positive attitude towards further study in mathematics and the use of mathematics in employment
- deliver in-depth study of mathematical concepts and the ways in which mathematics describes our world
- allow learners to interpret, communicate and manage information in mathematical form; skills which are vital to scientific and technological research and development
- deepen the learner's skills in using mathematical language and exploring advanced mathematical ideas

Progression into this Course

Entry to this Course is at the discretion of the centre. However, learners would normally be expected to have attained the skills and knowledge required by one or more of the following or by equivalent qualifications and/or experience:

- National 5 Mathematics Course

Skills, knowledge and understanding covered in the Course

This section provides further advice and guidance about skills, knowledge and understanding that could be included in the Course.

Note: teachers and lecturers should refer to the *Course Assessment Specification* for mandatory information about the skills, knowledge and understanding to be covered in this Course.

Mathematical skills are developed in each of the Course Units. An overview of the Units in which they are developed is shown below.

Mathematical skills	**Expressions and Functions**	**Relationships and Calculus**	**Applications**	**Course Assessment**
Interpret a situation which requires the use of mathematics and select an appropriate strategy	✓	✓	✓	✓
Explain a solution and, where appropriate, relate it to context	✓	✓	✓	✓
Use algebraic skills	✓	✓	✓	✓
Use geometric skills	✓		✓	✓
Use trigonometric skills	✓	✓		✓
Use calculus skills		✓	✓	✓

Many of these skills are relevant to learning in other curricular subjects such as geography, science and technology. In addition, they provide a useful foundation for vocational areas such as engineering, architecture, construction and medicine.

The table in Appendix 1 details the skills, knowledge and understanding of the Course and provides some examples of the contexts in which the mathematical skills can be developed.

Progression from this Course

This Course or its components may provide progression to:

- Advanced Higher Mathematics
- Advanced Higher Mathematics of Mechanics
- Advanced Higher Statistics

Mathematics has applications in many subject areas, and skills developed in this Course could support progression in this and other curriculum areas and employment.

Hierarchies

Hierarchy is the term used to describe Courses and Units which form a structured sequence involving two or more SCQF levels.

It is important that any content in a Course and/or Unit at one particular SCQF level is not repeated if a learner progresses to the next level of the hierarchy. The skills and knowledge should be able to be applied to new content and contexts to enrich the learning experience. This is for centres to manage.

The Higher Mathematics Course is in a hierarchy with the National 5 Mathematics Course.

National 5 Mathematics	**Higher Mathematics**
Expressions and Formulae	Expressions and Functions
Relationships	Relationships and Calculus
Applications	Applications
Course Assessment	Course Assessment

This hierarchical structure provides progression, aims to provide a mechanism for fall back, and enables learners to be given recognition for their best achievement. For example, achievement of the Units at Higher but not the Course assessment would provide fall back to Mathematics at National 5. The learner would only need to complete the Course assessment at National 5 to be given credit for the National 5 Course.

Approaches to learning and teaching

The purpose of this section is to provide general advice and guidance on approaches to learning and teaching across the Course.

The overall aim of the Course is to develop a range of mathematical operational and reasoning skills that can be used to solve mathematical and real-life problems. Approaches to learning and teaching should be engaging, with opportunities for personalisation and choice built in where possible.

A rich and supportive learning environment should be provided to enable a learner to achieve the best they can. This could include learning and teaching approaches such as:

- project-based tasks such as investigating the graphs of related functions, which could include using calculators or other technologies
- a mix of collaborative, co-operative or independent tasks which engage learners, for example by using differentiation to explore areas of science
- using materials available from service providers and authorities, eg working with a trigonometric model to predict the time of high tide in a harbour
- problem solving and critical thinking
- explaining thinking and presenting strategies and solutions to others such as discussing appropriate methods of solving trigonometric equations, perhaps using double angle formulae, and interpreting the solution set
- effective use of questioning and discussion to engage learners in explaining their thinking and checking their understanding of fundamental concepts
- making links in themes which cut across the curriculum to encourage transferability of skills, knowledge and understanding — including with technology, geography, sciences, social subjects and health and wellbeing. For example, using physics formulae and the application of calculus to the equations of motion under constant acceleration a, from initial speed u at position $x = 0$ and time $t = 0$ (for motion in a straight line):

  Given $a = \frac{dv}{dt}$ integrate to get $v = u + at$

  then note $v = \frac{ds}{dt}$ integrate to get $s = ut + \frac{1}{2}at^2$

  Sketch graphs of a, v and s versus t, and confirm the relationships using gradients and areas.

Technology could be used where appropriate to extend experience and confidence.

The development of mathematical skills is an active and productive process, building on learner's current knowledge, understanding and capabilities. Existing knowledge should form the starting point for any learning and teaching situation with new knowledge being linked to existing knowledge and built on. Presenting learners with an investigative or practical task is a useful way of allowing learners

to appreciate how a new idea relates to their existing knowledge and understanding.

Exposition by the teacher is an important technique. However, learners should be engaged as much as possible in all learning and teaching approaches. The engagement of learners could be enhanced by teachers/lecturers providing opportunities for personalisation and choice where appropriate.

Questions could be used to ascertain a learner's level of understanding and provide a basis for consolidation or remediation where necessary. Examples of probing questions could include:

1 How did you decide what to do?
2 How did you approach exploring and solving this task or problem?
3 Could this task or problem have been solved in a different way? If yes, what would you have done differently?

As learners develop concepts in mathematics, they will benefit from continual reinforcement and consolidation to build a foundation for progression.

Use of technology
Calculators with mathematical and graphical facilities and those with computer algebra systems (CAS) can be utilised as powerful tools both for processing data, especially in the study of statistics, and for reinforcing mathematical concepts. The use of such calculators should help learners gain confidence in making conjectures based on numerical or graphical evidence. Learners should be aware that errors are inevitably introduced in the course of computation or in the limitations of the graphical display. Learners should be aware of the limitations of technology and of the importance of always applying the strategy of checking.

Computers can also make a significant contribution to learning and teaching. The use of software packages will enhance the learning and teaching and allow learners greater flexibility through ease of computation and display. It is envisaged that increased availability and advances in technology will have a continuing and increasing influence in approaches to learning and teaching at this level.

More specific examples of suggested approaches to teaching and learning are detailed in the *Unit Support Notes*.

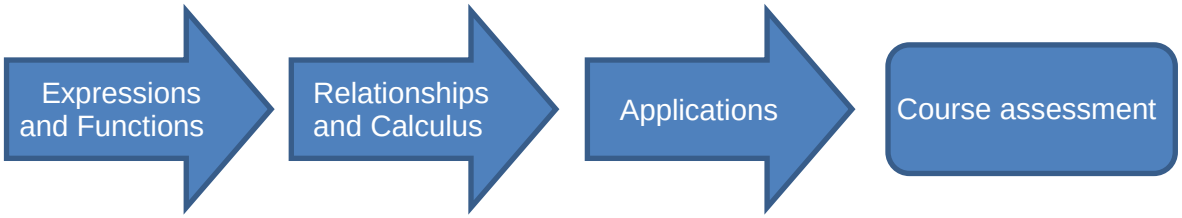
Sequencing and integration of Units within the Course
Sequencing and integration of the delivery and assessment of the Units within the Course is at the discretion of the centre.

The examples which follow illustrate possible approaches which may be adopted. Please note that other combinations are also possible.

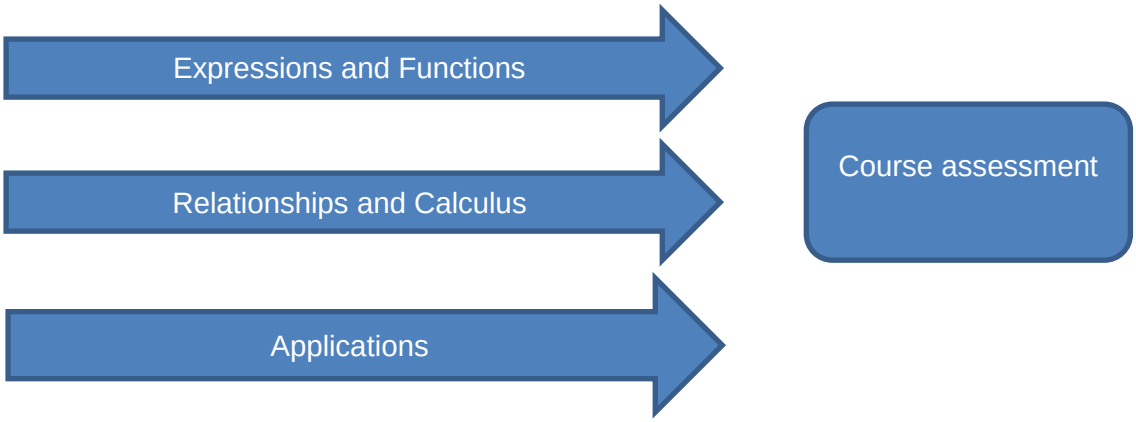
In these and other possible models of delivery, the development of numeracy skills should take place naturally in the learning and teaching of these Units. This should be recognised as contributing to the overall numeracy skills of the learner.

Example 1:
This example shows the possibility of delivering the *Expressions and Functions* Unit, the *Relationships and Calculus* Unit and the *Applications* Unit sequentially. This sequence could allow for the development of skills associated with

expressions and functions to be applied in relationships, both of which could then be applied and reinforced in the *Applications* Unit. Completion of all three Units would lead on to the Course assessment which draws on the skills, knowledge and understanding from across the Course.



Example 2:
This example shows the possibility of delivering all three Units — *Expressions and Functions*, *Relationships and Calculus* and *Applications* — concurrently. This approach may be suitable if learning and teaching is organised by grouping skills, for example, combining algebraic skills; combining geometric or trigonometric skills; or combining calculus skills. This approach has the potential of maximising the relevance and transferability of learning and teaching. Completion of all three Units leads to the Course assessment which draws on the skills, knowledge and understanding from across the Course.



Developing skills for learning, skills for life and skills for work

Learners are expected to develop broad generic skills as an integral part of their learning experience. The *Course Specification* lists the skills for learning, skills for life and skills for work that learners should develop through this Course. These are based on SQA's *Skills Framework: Skills for Learning, Skills for Life and Skills for Work* and must be built into the Course where there are appropriate opportunities. The level of these skills will be appropriate to the level of the Course. The following skills for learning, skills for life and skills for work are developed in this Course:

2 Numeracy

2.1 Number processes
2.2 Money, time and measurement
2.3 Information handling

5 Thinking skills

5.3 Applying
5.4 Analysing and evaluating

It is suggested that opportunities for developing the above skills for learning, skills for life and skills for work are built into learning and teaching wherever possible. The Higher Mathematics Course overtakes and builds on the numeracy skills, knowledge and understanding contained within the Numeracy Unit at National 5.

During the delivery of the Course there will also be opportunities for learners to develop their literacy skills and employability skills.

Literacy skills are particularly important as these skills allow learners to access, engage in and understand their learning; and to communicate their thoughts, ideas and opinions. This Course will provide learners with the opportunity to develop their literacy skills by analysing real-life contexts and communicating their thinking by presenting mathematical information in a variety of ways. This could include the use of numbers, formulae, diagrams, graphs, symbols and words.

Employability skills are the personal qualities, skills, knowledge, understanding, and attitudes required in changing economic environments. The mathematical operational and reasoning skills developed in this Course aim to enable learners to confidently respond to mathematical situations that can arise in the workplace. It aims to achieve this by providing learners with the opportunity to analyse a situation, decide which mathematical strategies to apply, work through those strategies effectively, and make informed decisions based on the results.

Further guidance on the development of skills for learning, skills for life and skills for work can be found in the *Unit Support Notes*.

Approaches to assessment

General guidance on assessment
A wide variety of approaches can be used to assess learners and gather evidence in the Higher Mathematics Course. The examples given here are not exhaustive.

Assessments must be valid, reliable and fit for purpose for the subject and the level and should fit with the learning and teaching approaches adopted.

Each assessment should therefore:

- be designed to allow learners to produce evidence to show they have achieved the required skills, knowledge and understanding for the Unit or Outcomes being assessed
- allow consistent judgements to be made by all assessors
- be appropriate for the Outcomes and the Assessment Standards in the Unit

Combining assessment across Units
When the Units are delivered as part of a Course, the assessment of Units can be combined.

The pattern of combined assessment can mirror that for integrated delivery as suggested in models shown in the section on 'Approaches to learning and teaching'.

A combined approach to assessment has the advantage of:

- enriching the assessment process for both learners and teachers/lecturers by bringing together elements of different Units
- avoiding duplication of assessment
- making learning and assessment more coherent and relevant for learners

Suggested approaches to assessment
The skills-based focus of the Course readily lends itself to a variety of approaches to assessment.

The following table gives some suggested approaches to assessment and examples of how they could be used to combine assessment across the Course.

Approach to assessment	Examples of approaches to assessment
Project/ investigation	Mathematical investigations are fundamental to the study of mathematics and often lead to an appreciation of how mathematics can be applied to solve problems in a broad range of fields such as STEM. Investigations usually develop from an initial problem which should be clearly stated and contain no ambiguity. At this level, investigations could: ♦ provide challenge and the opportunity for creativity ♦ contain multi-solution paths, allowing learners to choose different courses of action from a range of options
Problem solving tasks or activities	Problem solving tasks could be used to combine elements of mathematics within or across Units. Learners could be asked to identify possible solutions to solve problems in context and to communicate their solution appropriately. Mathematical modelling usually requires an experimental approach to problem solving. By considering alternatives, learners can use modelling to identify possible solutions to problems.
Short/extended response tests	The use of short answer/extended response tests may be appropriate for the combined assessment of skills. This type of assessment could be used to assess a learner's ability to apply mathematics with/without the aid of a calculator.

Whatever assessment approach is used, teachers/lecturers are encouraged to ensure that they are in line with guidance provided in the 'Equality and inclusion' section of this document.

Further guidance on approaches to assessment and gathering evidence for the Units can be found in the *Unit Support Notes*.

Exemplification of assessment is provided in Unit assessment support.

Gathering evidence
Evidence for assessment purposes could take a variety of forms such as:

- written evidence including calculations and graphics generated during supervised class work or discrete mathematical tests
- oral evidence arising from discussion between learners and the teacher/lecturer which shows learner ability and understanding across the Assessment Standard
- computer-generated assessment records or printouts from simulations
- photographs of project or investigative work
- a product such as a spreadsheet or computer-generated graphic

This list is not exhaustive and other types of evidence are also possible.

Achievement is on a pass/fail basis for the Outcomes. Learners who fail to achieve all of the Assessment Standards within the Outcomes will only need to be re-assessed on those Assessment Standards not achieved. Re-assessment should only follow after further work or remediation has been undertaken. Centres may consider it appropriate to delay re-assessment until further learning has taken place. Opportunities may exist for this by building it into other tasks within or across Units.

Authentication
Assessment should be carried out under supervision. For guidance on authentication of evidence which is gathered outwith the direct supervision of the teacher/lecturer responsible for the learner, eg outside the school or classroom, refer to SQA's *Guide to Assessment*.

Preparation for Course assessment

Each Course has additional time which may be used at the discretion of the teacher or lecturer to enable learners to prepare for Course assessment. This time may be used near the start of the Course and at various points throughout the Course for consolidation and support. It may also be used for preparation for Unit assessment, and towards the end of the Course, for further integration, revision and preparation and/or gathering evidence for Course assessment.

Information given in the *Course Specification* and the *Course Assessment Specification* about the assessment of added value is mandatory.

Courses from National 4 to Advanced Higher include assessment of added value. At Higher the added value will be assessed in the Course assessment.

The *Course Assessment Specification* addresses the key purposes and aims of the Course as defined in the Course Rationale.

In this Course, the Course assessment will focus on breadth, challenge and application. The learner will draw on and extend the skills they have learned during the Course.

In preparation for the Course assessment, it is recommended that learners are the given the opportunity to:

- develop mathematical operational and reasoning skills beyond the minimum competence required for the Units (see Appendix 1)
- integrate mathematical operational skills developed across the Units
- apply mathematical skills without the aid of a calculator to demonstrate an underlying grasp of mathematical concepts and processes

To achieve success in the Course, learners must show that they can apply knowledge and skills acquired across the Course to unseen situations.

There are two question papers, requiring learners to demonstrate aspects of breadth, challenge and application in mathematical contexts. In one of the question papers, the use of a calculator will be permitted. Learners will apply breadth and depth of knowledge and skills from across the Units to answer appropriately challenging questions.

The question papers will assess a selection of knowledge and skills acquired in the Course and will provide opportunities to apply skills in a wide range of situations, some of which may be new to the learner.

Equality and inclusion

At all times, teachers/lecturers should use inclusive approaches to assessment, taking into account the needs and experiences of their learners.

If a learner has a disability that affects their engagement in learning or ability to generate evidence for the Course, the centre could offer support.

It is recognised that centres have their own duties under equality and other legislation and policy initiatives. The guidance given in these *Course Support Notes* is designed to sit alongside these duties but is specific to the delivery and assessment of the Course.

It is important that centres are aware of and understand SQA's assessment arrangements for disabled learners, and those with additional support needs, when making requests for adjustments to published assessment arrangements. Centres will find more guidance on this in the series of publications on Assessment Arrangements on SQA's website: www.sqa.org.uk/sqa/14977.html.

Appendix 1: Content and learning and teaching notes

The first column refers to broad skills areas.

The second column is the mandatory skills, knowledge and understanding given in the *Course Assessment Specification*. This includes a description of the Unit standard and the added value for the Course assessment. Skills which could be sampled to confirm that learners meet the minimum competence of the Assessment Standards are indicated by a diamond bullet point. Those skills marked by an arrow bullet point are considered to be beyond minimum competence for the Units, but are part of the added value for the Course Assessment.

The third column gives suggested learning and teaching contexts to exemplify possible approaches to learning and teaching. These also provide examples of where the skills could be used in activities.

Mathematics (Higher) Expressions and Functions		
Operational skills		
Applying algebraic skills to logarithms and exponentials		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Manipulating algebraic expressions	♦ Simplifying an expression, using the laws of logarithms and exponents ♦ Solving logarithmic and exponential equations ♦ Using the laws of logarithms and exponents ➢ Solve for a and b equations of the following forms, given two pairs of corresponding values of x and y: $\log y = b\log x + \log a,\ y = ax^b$ and, $\log y = x\log b + \log a,\ y = ab^x$ ➢ Use a straight line graph to confirm relationships of the form $y = ax^b,\ y = ab^x$ ➢ Model mathematically situations involving the logarithmic or exponential function	Link logarithmic scale to science applications, eg decibel scale for sound, Richter scale of earthquake magnitude, astronomical scale of stellar brightness, acidity and pH in chemistry and biology. Note link between scientific notation and logs to base 10. Real-life contexts involving logarithmic and exponential characteristics, eg rate of growth of bacteria, calculations of money earned at various interest rates over time, decay rates of radioactive materials.
Applying trigonometric skills to manipulating expressions		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Manipulating trigonometric expressions	Application of: ♦ the addition or double angle formulae ♦ trigonometric identities Convert $a\cos x + b\sin x$ to $k\cos(x \pm \alpha)$ or $k\sin(x \pm \alpha),\ k > 0$ ♦ α in 1st quadrant ➢ α in any quadrant	Learners can be shown how formulae for $\cos(\alpha + \beta)$ and $\sin(\alpha + \beta)$ can be used to prove formulae for $\sin 2\alpha$, $\cos 2\alpha$, $\tan(\alpha + \beta)$. Emphasise the distinction between $\sin x°$ and $\sin x$ (degrees and radians). Learners should be given practice in applying the standard formulae, eg expand $\sin 3x$ or $\cos 4x$. Learners should be exposed to geometric problems which require the use of addition or double angle formulae. Example of use in science: a train of moving water waves of wavelength λ has a profile $y = H\sin\left\{2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]\right\}$

Applying algebraic and trigonometric skills to functions		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Identifying and sketching related functions	♦ Identifying a function from a graph, or sketching a function after a transformation of the form $kf(x),\ f(kx),\ f(x)+k,\ f(x+k)$ or a combination of these ➢ Sketch $y=f'(x)$ given the graph of $y=f(x)$ ➢ Sketch the inverse of a logarithmic or an exponential function ➢ Completing the square in a quadratic expression where the coefficient of x^2 is non-unitary	Use of graphic calculators here to explore various transformations. Learners should be able to recognise a function from its graph. Interpret formulae/equations for maximum/minimum values and when they occur.
Determining composite and inverse functions	♦ Determining a composite function given $f(x)$ and $g(x)$, where $f(x)$, $g(x)$ can be trigonometric, logarithmic, exponential or algebraic functions — including basic knowledge of domain and range ♦ $f^{-1}(x)$ of functions ➢ Know and use the terms domain and range	$f(g(x))$ where $f(x)$ is a trigonometric function/logarithmic function and $g(x)$ is a polynomial. Learners should be aware that $f(g(x))=x$ implies $f(x)$ and $g(x)$ are inverses.
Applying geometric skills to vectors		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Determining vector connections	♦ Determining the resultant of vector pathways in three dimensions ♦ Working with collinearity ♦ Determining the coordinates of an internal division point of a line	Learners should work with vectors in both two and three dimensions. In order to 'show' collinearity, communication should include mention of parallel vectors and 'common point'. Distinction made between writing in coordinate and component form.
Working with vectors	♦ Evaluate a scalar product given suitable information and determining the angle between two vectors ➢ Apply properties of the scalar product	Also, introduce the zero vector. Perpendicular and distributive properties of vectors should be investigated, eg If $\lvert\mathbf{a}\rvert, \lvert\mathbf{b}\rvert \neq 0$ then $\mathbf{a}\cdot\mathbf{b}=0$ if and only if

	➢ Using and finding unit vectors including $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ as a basis	the directions of $\mathbf{a}$ and $\mathbf{b}$ are at right angles. Example of broader application: sketch a vector diagram of the three forces on a kite, when stationary: its weight, force from the wind (assume normal to centre of kite inclined facing the breeze) and its tethering string. These must sum to zero.
Reasoning skills		
Interpreting a situation where mathematics can be used and identifying a valid strategy	Can be attached to a skill of Outcome 1 to require analysis of a situation.	This should be a mathematical or real-life context problem in which some analysis is required. The learner should be required to choose an appropriate strategy and employ mathematics to the situation.
Explaining a solution and, where appropriate, relating it to context	Can be attached to a skill of Outcome 1 to require explanation of the solution given.	The learner should be required to give meaning to the determined solution in everyday language.

Mathematics (Higher) Relationships and Calculus		
Operational skills		
Applying algebraic skills to solve equations		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Solving algebraic equations	♦ Factorising a cubic polynomial expression with unitary x^3 coefficient ➢ Factorising a cubic or quartic polynomial expression with non-unitary coefficient of the highest power Solving polynomial equations: ♦ Cubic with unitary x^3 coefficient ➢ Cubic or quartic with non-unitary coefficient of the highest power Discriminant: ♦ Given the nature of the roots of an equation, use the discriminant to find an unknown ➢ Solve quadratic inequalities, $ax^2+bx+c \geq 0$ (or ≤ 0) Intersection: ➢ Finding the coordinates of the point(s) of intersection of a straight line and a curve or of two curves	Strategies for factorising polynomials, ie synthetic division, inspection, algebraic long division. Factorising quadratic at National 5 or in previous learning led to solution(s) which learners can link to graph of function. Factorising polynomials beyond degree 2 allows extension of this concept. Identifying when an expression is not a polynomial (negative/fractional powers). Recognise repeated root is also a stationary point. Emphasise meaning of solving $f(x)=g(x)$. Learners should encounter the Remainder Theorem and how this leads to the fact that for a polynomial equation, $f(x)=0$, if $(x-h)$ is a factor of $f(x)$, h is a root of the equation and vice versa. Learners' communication should include a statement such as 'since $f(h)=0$' or 'since remainder is 0'. Learners should also experience divisors/factors of the form $(ax-b)$. As far as possible, solutions of algebraic equations should be linked to a graph of function(s), with learners encouraged to make such connections. (Use of graphic calculators/refer to diagram in question/ sketch diagrams to check solutions.)

Applying trigonometric skills to solve equations		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Solving trigonometric equations	♦ Solve trigonometric equations in degrees, involving trigonometric formulae, in a given interval ➢ Solving trigonometric equations in degrees or radians, including those involving the wave function or trigonometric formulae or identities, in a given interval	Link to trigonometry of Expressions and Functions Unit. Real-life contexts should be used whenever possible. Solution of trigonometric equations could be introduced graphically. Recognise when a solution should be given in radians (eg $0 \leq x \leq \pi$). In the absence of a degree symbol, radians should be used. A possible application is the refraction of a thin light beam passing from air into glass. Its direction of travel is bent towards the line normal to the surface, according to Snell's law.
Applying calculus skills of differentiation		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Differentiating functions	♦ Differentiating an algebraic function which is, or can be simplified to, an expression in powers of x ♦ Differentiating $k\sin x$, $k\cos x$ ➢ Differentiating a composite function using the chain rule	Examples from science using the terms associated with rates of change, eg acceleration, velocity.
Using differentiation to investigate the nature and properties of functions	♦ Determining the equation of a tangent to a curve at a given point by differentiation ➢ Determining where a function is strictly increasing/decreasing ➢ Sketching the graph of an algebraic function by determining stationary points and their nature as well as intersections with the axes and behaviour of $f(x)$ for large positive and negative values of x	Learners should know that the gradient of a curve at a point is defined to be the gradient of the tangent to the curve at that point. Learners should know when a function is either strictly increasing, decreasing or has a stationary value, and the conditions for these. The second derivative or a detailed nature table can be used. Stationary points should include horizontal points of inflexion.

Applying calculus skills of integration		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Integrating functions	♦ Integrating an algebraic function which is, or can be, simplified to an expression of powers of x ♦ Integrating functions of the form $f(x)=(x+q)^n,\ n\neq -1$ ♦ Integrating functions of the form $f(x)=p\cos x$ and $f(x)=p\sin x$ ➢ Integrating functions of the form $f(x)=(px+q)^n,\ n\neq -1$ ➢ Integrating functions of the form $f(x)=p\cos(qx+r)$ and $p\sin(qx+r)$ ➢ Solving differential equations of the form $\frac{dy}{dx}=f(x)$	Know the meaning of the terms integral, integrate, constant of integration, definite integral, limits of integration, indefinite integral, area under a curve. Know that if $f(x)=F'(x)$ then $\int_a^b f(x)dx=F(b)-F(a)$ and $\int f(x)dx=F(x)+C$ where C is the constant of integration. Could be introduced by anti-differentiation. Learners should experience integration of $\cos^2 x$ and $\sin^2 x$ using $\cos^2 x=\frac{1}{2}(1+\cos 2x)$ $\sin^2 x=\frac{1}{2}(1-\cos 2x)$
Using integration to calculate definite integrals	♦ Calculating definite integrals of polynomial functions with integer limits ➢ Calculating definite integrals of functions with limits which are integers, radians, surds or fractions	Extend to area beneath the curve between the limits.
Reasoning skills		
Interpreting a situation where mathematics can be used and identifying a valid strategy	Can be attached to a skill of Outcome 1 to require analysis of a situation.	This should be a mathematical or real-life context problem in which some analysis is required. The learner should be required to choose an appropriate strategy and employ mathematics to the situation.
Explaining a solution and, where appropriate, relating it to context	Can be attached to a skill of Outcome 1 to require explanation of the solution given.	The learner should be required to give meaning to the determined solution in everyday language.

Mathematics (Higher) Applications		
Operational skills		
Applying algebraic skills to rectilinear shapes		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Applying algebraic skills to rectilinear shapes	♦ Finding the equation of a line parallel to and a line perpendicular to a given line ♦ Using $m = \tan\theta$ to calculate a gradient or angle ➢ Using properties of medians, altitudes and perpendicular bisectors in problems involving the equation of a line and intersection of lines ➢ Determine whether or not two lines are perpendicular	Emphasise the 'gradient properties' of $m_1 = m_2$ and $m_1 m_2 = -1$. Use practical contexts for triangle work where possible. Emphasise differences in median, altitude etc. Perhaps investigate properties and intersections. Avoid approximating gradients to decimals. Knowledge of the basic properties of triangles and quadrilaterals would be useful. In order to 'show' collinearity, statement should include mention of 'common point', eg since $m_{AB} = m_{BC}$ and B is a common point. Understanding of terms such as orthocentre, circumcentre and concurrency.
Applying algebraic skills to circles		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Applying algebraic skills to circles	♦ Determining and using the equation of a circle ♦ Using properties of tangency in the solution of a problem ➢ Determining the intersection of circles or a line and a circle	Link to work on discriminant (one point of contact). Develop equation of circle (centre the origin) from Pythagoras, and extend this to circle with centre (a,b) or relate to transformations. Demonstrate application of discriminant. Learners made aware of different ways in which more than one circle can be positioned, eg intersecting at one/two/no points, sharing same centre (concentric), one circle inside another. Practice in applying knowledge of geometric properties of circles in finding related points (eg stepping out method). Solutions should not be obtained from scale drawings.

Applying algebraic skills to sequences		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Modelling situations using sequences	♦ Determining a recurrence relation from given information and using it to calculate a required term ♦ Finding and interpreting the limit of a sequence, where it exists	Where possible, use examples from real-life situations such as where concentrations of chemicals/medicines are important.
Applying calculus skills to optimisation and area		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Applying differential calculus	♦ Determining the optimal solution for a given problem ➢ Determine the greatest/least values of a function on a closed interval ➢ Solving problems using rate of change	Max/min problems applied in context, eg minimum amount of card for creating a box, maximum output from machines. Rate of change linked to science. Optimisation in science, eg an aeroplane cruising at speed v at a steady height, has to use power to push air downwards to counter the force of gravity, and to overcome air resistance to sustain its speed. The energy cost per km of travel is given approximately by: $E = Av^2 + Bv^2$ (A and B depend on the size and weight of the plane). At the optimum speed $\frac{dE}{dv} = 0$, thus get an expression for v_{opt} in terms of A and B.
Applying integral calculus	♦ Finding the area between a curve and the x-axis ♦ Finding the area between a straight line and a curve or two curves ➢ Determine and use a function from a given rate of change and initial conditions	Develop from Relationships and Calculus Unit. Use of graphical calculators for an investigative approach. Area between curves by subtraction of individual areas — use of diagrams, graphing packages. Reducing area to be determined to smaller components in order to estimate segment of area between curve and x-axis. Use of area formulae (triangle/rectangle) in solving such problems. A practical application of the integral of $\frac{1}{x^2}$ is to calculate

		the energy required to lift an object from the earth's surface into space. The work energy required is $E = \int F dr$ where F is the force due to the earth's gravity and r is the distance from the centre of the earth. For a 1 kg object $E = -\int \left(\frac{GM}{r^2} \right) dr$ where M is the mass of the earth and G is the universal gravitational constant. $GM = 4 \cdot 0 \times 10^{14}\ \mathrm{m^3\,s^{-2}}$. The integration extends from $r = 6 \cdot 4 \times 10^{6}\ \mathrm{m}$ (the radius of the earth) to infinity.
Reasoning skills		
Interpreting a situation where mathematics can be used and identifying a valid strategy	Can be attached to a skill of Outcome 1 to require analysis of a situation.	This should be a mathematical or real-life context problem in which some analysis is required. The learner should be required to choose an appropriate strategy and employ mathematics to the situation.
Explaining a solution and, where appropriate, relating it to context	Can be attached to a skill of Outcome 1 to require explanation of the solution given.	The learner should be required to give meaning to the determined solution in everyday language.
Additional Information		
Symbols, terms and sets: the symbols: ∈, ∉, $\{\ \}$ {} the terms: set, subset, empty set, member, element the conventions for representing sets, namely: $\mathbb{N}$, the set of natural numbers, $\{1, 2, 3, \ldots\}$ $\mathbb{W}$, the set of whole numbers, $\{0, 1, 2, 3, \ldots\}$ $\mathbb{Z}$, the set of integers $\mathbb{Q}$, the set of rational numbers $\mathbb{R}$, the set of real numbers The content listed above is not examinable but learners are expected to be able to understand its use.		

Appendix 2: Reference documents

The following reference documents will provide useful information and background.

- Assessment Arrangements (for disabled learners and/or those with additional support needs) — various publications on SQA's website: www.sqa.org.uk/sqa//14977.html.
- *Building the Curriculum 4: Skills for learning, skills for life and skills for work*
- *Building the Curriculum 5: A framework for assessment*
- Course Specifications
- Design Principles for National Courses
- *Guide to Assessment* (June 2008)
- *Principles and practice papers for curriculum areas*
- *Research Report 4 — Less is More: Good Practice in Reducing Assessment Time*
- *Coursework Authenticity — a Guide for Teachers and Lecturers*
- *SCQF Handbook: User Guide* (published 2009) and SCQF level descriptors (reviewed during 2011 to 2012): www.sqa.org.uk/sqa/4595.html
- *SQA Skills Framework: Skills for Learning, Skills for Life and Skills for Work*
- SQA Guidelines on e-assessment for Schools
- SQA Guidelines on Online Assessment for Further Education
- SQA e-assessment web page: www.sqa.org.uk/sqa/5606.html

Administrative information

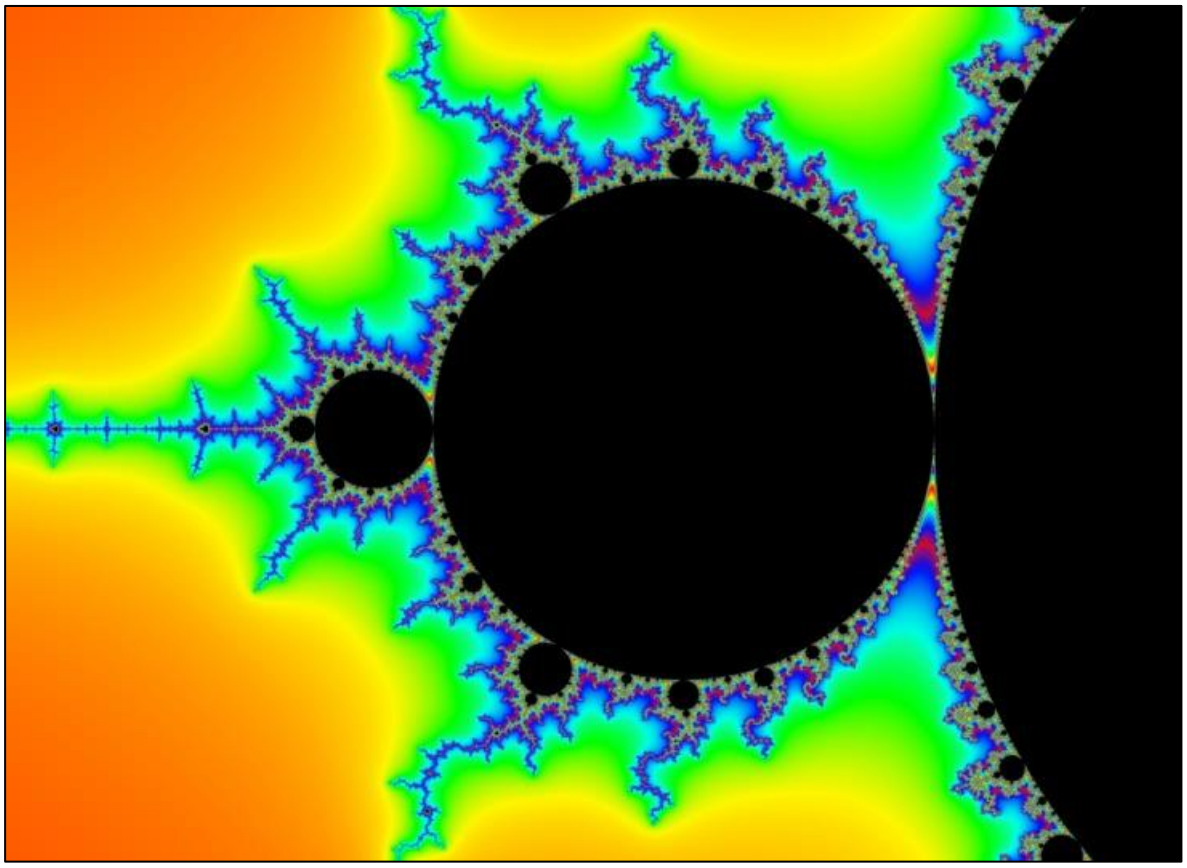
Published: May 2016 (version 2.2)

History of changes to Course Support Notes

Version	Description of change	Authorised by	Date
2.0	Content amended and expanded in Appendix 1. Mathematical skill tables amended to be in line with Unit Specifications.	Qualifications Development Manager	June 2014
2.1	Appendix 1: pages 15 and 16 — typographical errors corrected; page 18 — bullet point removed.	Qualifications Manager	August 2014
2.2	Appendix 1: Content and learning and teaching notes — minor clarifications: 'Identifying or sketching a function' changed to Identifying a function from a graph, or sketching a function'; 'Using unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ as a basis' changed to 'Using and finding unit vectors including $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ as a basis'; 'angle between 2 vectors' changed to 'angle between two vectors'; 'n not equal to -1' changed to '$n \neq -1$ '; 'W' changed to '$\mathbb{W}$'.	Qualifications Manager	May 2016

Unit Support Notes

Unit Support Notes — Mathematics: Expressions and Functions (Higher)



Please refer to the note of changes at the end of this document for details of changes from previous version (where applicable).

Introduction

These support notes are not mandatory. They provide advice and guidance on approaches to delivering and assessing the *Expressions and Functions* (Higher) Unit. They are intended for teachers and lecturers who are delivering this Unit. They should be read in conjunction with:

- the *Unit Specification*
- the *Course Specification*
- the *Course Assessment Specification*
- the *Course Support Notes*
- appropriate assessment support materials

General guidance on the Unit

Aims

The *Expressions and Functions* (Higher) Unit is a mandatory Unit in the Higher Mathematics Course. The *Expressions and Functions* Unit is also available as a free-standing Unit and is designed to meet the needs of a broad range of learners who may choose to study it.

The general aim of this Unit is to develop knowledge and skills that involve the manipulation of expressions, the use of vectors and the study of mathematical functions. The Outcomes cover aspects of algebra, geometry and trigonometry, and also skills in mathematical reasoning and modelling.

Progression into this Unit

Entry into this Unit is at the discretion of the centre. However, learners would normally be expected to have attained the skills, knowledge and understanding required by one or more of the following or equivalent qualifications and/or experience:

- National 5 Mathematics Course or its component Units

Skills, knowledge and understanding covered in the Unit

Information about skills, knowledge and understanding is given in the Higher Mathematics *Course Support Notes*.

If this Unit is being delivered on a free-standing basis, teachers and lecturers are free to select the skills, knowledge, understanding and contexts which are most appropriate for delivery in their centres.

Progression from this Unit

This Unit may provide progression to:

- Other Units within Higher Mathematics

Mathematics has applications in many subject areas, and skills developed in this Course could support progression in this and other curriculum areas and employment.

Approaches to learning and teaching

The purpose of this section is to provide advice and guidance on the sequencing and integration of approaches to learning and teaching for this Unit.

Sequencing
The skills linked to the *Expressions and Functions* Unit can be delivered sequentially or concurrently. Teachers/lecturers can choose to deliver these in any order. There is no specific amount of time allocated to each. This will often depend on the needs of the learners.

Integration
Combining skills within Units
Outcomes could be integrated by combining the reasoning skills Outcome with any of the skills in Outcome 1.

- Algebraic or trigonometric expressions could be derived from a mathematical problem before simplification.
- The context of graphs could be discussed and interpretations made of related points.
- Vectors could be derived from a real-life situation before working with them.

Combining skills across Units
For centres delivering this Unit as part of the Higher Mathematics Course, the Outcomes of this Unit may be integrated with Outcomes in the other Units.

- Expressions of this Unit could be combined with equations of the *Relationships and Calculus* Unit.
- The graphs of functions of this Unit could be combined with the equations of the *Relationships and Calculus* Unit.

The Higher Mathematics *Course Support Notes* provide further advice and guidance on possible approaches to learning and teaching which apply to all Units of the Course.

Developing skills for learning, skills for life and skills for work

For this Unit there are significant opportunities to develop the following skills for learning, skills for life and skills for work. Some of these opportunities are described in the table below:

Skills for Learning, Skills for Life and Skills for Work	Suggested Approaches for Learning and Teaching
Numeracy is the ability to use numbers to solve problems by counting, doing calculations, measuring, and understanding graphs and charts. This is also the ability to understand the results.	Throughout this Unit, learners will have ample opportunities to: use number to solve both real-life and STEM-related problems and work with information through analysis and interpretation, by drawing conclusions and making deductions and informed decisions.
Applying Applying is the ability to use existing information to solve a problem in a different context, and to plan, organise and complete a task.	Wherever possible, learners should be given the opportunity to apply the skills, knowledge and understanding they have developed to solve mathematical problems in a range of real-life, cross-curricular and STEM-related contexts. Learners could be encouraged to think about how they are going to tackle problems, decide which skills and processes to use and then carry out the processes to complete the task. To determine a learner's level of understanding, learners should be encouraged to show and explain their thinking at all times. At Higher, learners could be encouraged to think creatively to adapt strategies to suit the given problem or situation.
Analysing and evaluating This covers the ability to identify and weigh-up the features of a situation or issue and to use your judgement of them in coming to a conclusion. It includes reviewing and considering any potential solutions.	Wherever possible, learners could be given the opportunity to identify real-life tasks or STEM situations which require the use of mathematics. Learners should be encouraged to analyse the task or situation to decide how it can be addressed and what mathematical skills will need to be applied. Learners should also be provided with opportunities to interpret the results of their calculations and to draw conclusions. Conclusions drawn by the learner could be used to form the basis of a model for making future choices or decisions.

There may also be further opportunities for the development of additional skills for learning, skills for life and skills for work in the delivery of this Unit. These opportunities may vary and are at the discretion of the centre.

Approaches to assessment and gathering evidence

The purpose of this section is to give advice and guidance on approaches to integrating assessment within this Unit.

The *Expressions and Functions* Unit can be assessed in a variety of ways and could include for example:

- a project or investigation
- problem solving tasks or activities
- short/extended response tests

These approaches are not exhaustive and other possibilities also exist. The following table gives some examples of how these approaches could be used within the Unit to provide a varied and integrated assessment experience. This approach aims to make the assessment process more coherent and meaningful for learners.

The sequencing and integration of assessment for this Unit could also mirror the examples described in the section on 'Approaches to learning and teaching'.

Approach to assessment	**Examples of approaches to assessment**
Project/ investigation	Mathematical investigations are fundamental to the study of mathematics and often lead to an appreciation of how mathematics can be applied. Learners could carry out an investigation into the resultant of forces using vectors. This could be combined with using reasoning skills to interpret a real-life situation to form the vectors.
Problem solving tasks or activities	Learners could be given a discrete task or activity which would provide the opportunity to gather evidence. Learners could be asked to match functions to graphs from a jumbled group and to describe the relationship.
Short/extended response tests	Learners could be given a short answer or extended response test. This could be used to assess use of the double angle formulae or trigonometric identities.

It would normally be expected that considerable learning and teaching would have taken place prior to the collection of evidence for assessment purposes.

Further advice and guidance on assessment for the Mathematics Course and its components Units is contained within the *Course Support Notes.*

Exemplification of assessment is provided in Unit assessment support.

When delivering this Unit as part of the Higher Mathematics Course, reference should be made to the appropriate content statements within the 'Further mandatory information on Course coverage' section in the *Course Assessment Specification*.

Equality and inclusion

Information about equality and inclusion issues related to this and other Units in Mathematics is given in the Higher Mathematics *Course Support Notes*.

It is recognised that centres have their own duties under equality and other legislation and policy initiatives. The guidance given in these *Unit Support Notes* is designed to sit alongside these duties but is specific to the delivery and assessment of the Unit.

Alternative approaches to Unit assessment to take account of the specific needs of learners can be used. However, the centre must be satisfied that the integrity of the assessment is maintained and that alternative approaches to assessment will, in fact, generate the necessary evidence of achievement.

Appendix 1: Reference documents

The following reference documents will provide useful information and background.

- Assessment Arrangements (for disabled learners and/or those with additional support needs) — various publications on SQA's website: www.sqa.org.uk/sqa//14977.html.
- *Building the Curriculum 4: Skills for learning, skills for life and skills for work*
- *Building the Curriculum 5: A framework for assessment*
- Course Specifications
- Design Principles for National Courses
- *Guide to Assessment (June 2008)*
- *Principles and practice papers for curriculum areas*
- *Research Report 4 — Less is More: Good Practice in Reducing Assessment Time*
- *Coursework Authenticity — a Guide for Teachers and Lecturers*
- *SCQF Handbook: User Guide* (published 2009) and SCQF level descriptors (reviewed during 2011 to 2012): www.sqa.org.uk/sqa/4595.html
- *SQA Skills Framework: Skills for Learning, Skills for Life and Skills for Work*
- SQA Guidelines on e-assessment for Schools
- SQA Guidelines on Online Assessment for Further Education
- SQA e-assessment web page: www.sqa.org.uk/sqa/5606.html

Administrative information

Published: June 2014 (version 1.0)

History of changes to Unit Support Notes

Version	Description of change	Authorised by	Date

Unit Support Notes

Unit Support Notes — Mathematics: Relationships and Calculus (Higher)

Please refer to the note of changes at the end of this document for details of changes from previous version (where applicable).

Introduction

These support notes are not mandatory. They provide advice and guidance on approaches to delivering and assessing the *Relationships and Calculus* (Higher) Unit. They are intended for teachers and lecturers who are delivering this Unit. They should be read in conjunction with:

- the *Unit Specification*
- the *Course Specification*
- the *Course Assessment Specification*
- the *Course Support Notes*
- appropriate assessment support materials

General guidance on the Unit

Aims

The *Relationships and Calculus* (Higher) Unit is a mandatory Unit in the Higher Mathematics Course. The *Relationships and Calculus* Unit is also available as a free-standing Unit and is designed to meet the needs of a broad range of learners who may choose to study it.

The general aim of this Unit is to develop knowledge and skills that involve solving equations and to introduce both differential calculus and integral calculus. The Outcomes cover aspects of algebra, trigonometry, calculus, and also skills in mathematical reasoning and modelling.

Progression into this Unit

Entry into this Unit is at the discretion of the centre. However, learners would normally be expected to have attained the skills, knowledge and understanding required by one or more of the following or equivalent qualifications and/or experience:

- National 5 Mathematics Course or its component Units

Skills, knowledge and understanding covered in the Unit

Information about skills, knowledge and understanding is given in the Higher Mathematics *Course Support Notes*.

If this Unit is being delivered on a free-standing basis, teachers and lecturers are free to select the skills, knowledge, understanding and contexts which are most appropriate for delivery in their centres.

Progression from this Unit

This Unit may provide progression to:

- Other Units within Higher Mathematics

Mathematics has applications in many subject areas, and skills developed in this Course could support progression in this and other curriculum areas and employment.

Approaches to learning and teaching

The purpose of this section is to provide advice and guidance on the sequencing and integration of approaches to learning and teaching for this Unit.

Sequencing
The skills linked to the *Relationships and Calculus* Unit can be delivered sequentially or concurrently. Teachers/lecturers can choose to deliver these in any order. There is no specific amount of time allocated to each. This will often depend on the needs of the learners.

Integration

Combining skills within Units
Outcomes could be integrated by combining the reasoning skills Outcome with any of the skills in Outcome 1.

- The results of solving equations could be explained within a context.
- A tangency problem could be set in a science context, such as an object being held in a circular motion and then released.
- The value of definite integrals could be compared, particularly those in which the graphs cross the x-axis.

Combining skills across Units
For centres delivering this Unit as part of the Higher Mathematics Course, Outcomes of this Unit may be integrated with Outcomes in the other Units.

- Equations of this Unit with expressions of the *Expressions and Functions* Unit.
- Differential calculus of this Unit with optimisation of the *Applications* Unit.
- Integral calculus of this Unit with area of the *Applications* Unit.

The Higher Mathematics *Course Support Notes* provide further advice and guidance on approaches to learning and teaching which apply to all component Units of the Course.

Developing skills for learning, skills for life and skills for work

For this Unit there are significant opportunities to develop the following skills for learning, skills for life and skills for work. Some of these opportunities are described in the table below:

Skills for learning, skills for life and skills for work	Suggested approaches for learning and teaching
Numeracy is the ability to use numbers to solve problems by counting, doing calculations, measuring, and understanding graphs and charts. This is also the ability to understand the results.	Throughout this Unit, learners will have ample opportunities to use number to solve problems in real-life situations and other areas of the curriculum such as geography, IT, and STEM-related areas. Learners will have the opportunity to devise problem solving strategies and to handle information through analysis and interpretation, by drawing conclusions and making deductions and informed decisions.
Applying Applying is the ability to use existing information to solve a problem in a different context, and to plan, organise and complete a task.	Wherever possible, learners should be given the opportunity to apply the skills, knowledge and understanding they have developed to solve mathematical problems in a range of real-life, cross-curricular and STEM-related contexts. For example, learners could be encouraged to think about how they are going to tackle an engineering problem, decide which skills and processes to use and then carry out the processes to complete the task. Learners should be encouraged to show and explain their thinking at all times so that their level of understanding can be determined.' At Higher level, learners could be encouraged to think creatively to adapt strategies to suit the given problem or situation.
Analysing and evaluating This covers the ability to identify and weigh-up the features of a situation or issue and to use your judgement of them in coming to a conclusion. It includes reviewing and considering any potential solutions.	Wherever possible, learners could be given the opportunity to identify real-life tasks or STEM situations which require the use of mathematics. Learners should be encouraged to analyse the task or situation to decide how it can be addressed and what mathematical skills will need to be applied. Learners should also be provided with opportunities to interpret and evaluate the results of their calculations or mathematical strategies to draw conclusions. Conclusions drawn by the learner could be used to form the basis of a model for making future choices or decisions.

There may also be further opportunities for the development of additional skills for learning, skills for life and skills for work in the delivery of this Unit. These opportunities may vary and are at the discretion of the centre.

Approaches to assessment and gathering evidence

The purpose of this section is to give advice and guidance on approaches to integrating assessment within this Unit.

The *Relationships and Calculus* Unit can be assessed in a variety of ways and could include for example:

- a project or investigation
- problem solving tasks or activities
- short/extended response tests

These approaches are not exhaustive and other possibilities also exist.

The following table gives some examples of how these approaches could be used within the Unit to provide a varied and integrated assessment experience. This approach aims to make the assessment process more coherent and meaningful for learners.

The sequencing and integration of assessment for this Unit could also mirror the models described in the section on 'Approaches to learning and teaching'.

Approach to assessment	**Examples of approaches to assessment**
Project/ investigation	Mathematical investigations are fundamental to the study of mathematics and often lead to an appreciation of how mathematics can be applied. Learners could investigate the meaning and uses of the discriminant.
Problem solving tasks or activities	Learners could be given a discrete task or activity which would provide the opportunity to gather evidence. Learners could be asked to use given algebraic/ trigonometric models in real-life situations.
Short/extended response tests	Learners could be given a short answer or extended response test. A test could cover various types of trigonometric and algebraic equation.

It would normally be expected that considerable learning and teaching would have taken place prior to the collection of evidence for assessment purposes.

Further advice and guidance on assessment for the Mathematics Course and its components Units is contained within the *Course Support Notes.*

Exemplification of assessment is provided in Unit assessment support.

When delivering this Unit as part of the Higher Mathematics Course, reference should be made to the appropriate content statements within the 'Further mandatory information on Course coverage' section in the *Course Assessment Specification*.

Equality and inclusion

Information about equality and inclusion issues related to this and other Units in Mathematics is given in the Higher Mathematics *Course Support Notes*.

It is recognised that centres have their own duties under equality and other legislation and policy initiatives. The guidance given in these *Unit Support Notes* is designed to sit alongside these duties but is specific to the delivery and assessment of the Unit.

Alternative approaches to Unit assessment to take account of the specific needs of learners can be used. However, the centre must be satisfied that the integrity of the assessment is maintained and that alternative approaches to assessment will, in fact, generate the necessary evidence of achievement.

Appendix 1: Reference documents

The following reference documents will provide useful information and background.

- Assessment Arrangements (for disabled learners and/or those with additional support needs) — various publications on SQA's website: www.sqa.org.uk/sqa//14977.html.
- *Building the Curriculum 4: Skills for learning, skills for life and skills for work*
- *Building the Curriculum 5: A framework for assessment*
- Course Specifications
- Design Principles for National Courses
- *Guide to Assessment (June 2008)*
- *Principles and practice papers for curriculum areas*
- *Research Report 4 — Less is More: Good Practice in Reducing Assessment Time*
- *Coursework Authenticity — a Guide for Teachers and Lecturers*
- *SCQF Handbook: User Guide* (published 2009) and SCQF level descriptors (reviewed during 2011 to 2012): www.sqa.org.uk/sqa/4595.html
- *SQA Skills Framework: Skills for Learning, Skills for Life and Skills for Work*
- SQA Guidelines on e-assessment for Schools
- SQA Guidelines on Online Assessment for Further Education
- SQA e-assessment web page: www.sqa.org.uk/sqa/5606.html

Administrative information

Published: June 2014 (version 1.0)

History of changes to Unit Support Notes

Version	Description of change	Authorised by	Date

Unit Support Notes

Unit Support Notes — Mathematics: Applications (Higher)

Please refer to the note of changes at the end of this document for details of changes from previous version (where applicable).

Introduction

These support notes are not mandatory. They provide advice and guidance on approaches to delivering and assessing the *Applications* (Higher) Unit. They are intended for teachers and lecturers who are delivering this Unit. They should be read in conjunction with:

- the *Unit Specification*
- the *Course Specification*
- the *Course Assessment Specification*
- the *Course Support Notes*
- appropriate assessment support materials

General guidance on the Unit

Aims

The *Applications* (Higher) Unit is a mandatory Unit in the Higher Mathematics Course. The *Applications* Unit is also available as a free-standing Unit and is designed to meet the needs of a broad range of learners who may choose to study it.

The general aim of this Unit is to develop knowledge and skills that involve geometric applications, applications of sequences and applications of calculus. The Outcomes cover aspects of algebra, geometry, calculus, and also skills in mathematical reasoning and modelling.

Progression into this Unit

Entry into this Unit is at the discretion of the centre. However, learners would normally be expected to have attained the skills, knowledge and understanding required by one or more of the following or equivalent qualifications and/or experience:

- National 5 Mathematics Course or its component Units

Skills, knowledge and understanding covered in the Unit

Information about skills, knowledge and understanding is given in the Higher Mathematics *Course Support Notes*.

If this Unit is being delivered on a free-standing basis, teachers and lecturers are free to select the skills, knowledge, understanding and contexts which are most appropriate for delivery in their centres.

Progression from this Unit

This Unit may provide progression to:

- other Units within Higher Mathematics

Mathematics has applications in many subject areas, and skills developed in this Course could support progression in this and other curriculum areas and employment.

Approaches to learning and teaching

The purpose of this section is to provide advice and guidance on the sequencing and integration of approaches to learning and teaching for this Unit.

Sequencing
The skills linked to the *Applications* Unit can be delivered sequentially or concurrently. Teachers/lecturers can choose to deliver these in any order. There is no specific amount of time allocated to each. This will often depend on the needs of the learners.

Integration

Combining skills within Units
Outcomes could be integrated by combining the reasoning skills Outcome with any of the other Outcomes within this Unit.

- Equations can be interpreted/determined from geometrical diagrams.
- Recurrence relations can be determined from a real-life context.
- Problems of optimisation and area can be set from situations in science or technology.

Combining skills across Units
For centres delivering this Unit as part of the Mathematics Higher Course, Outcomes of this Unit may be integrated with Outcomes in the other Units.

- Optimisation of this Unit with differential calculus of the *Relationships and Calculus* Unit.
- Area of this Unit with integral calculus of the *Relationships and Calculus* Unit.

The Higher Mathematics *Course Support Notes* provide further advice and guidance on possible approaches to learning and teaching which apply to all component Units of the Course.

Developing skills for learning, skills for life and skills for work

For this Unit there are significant opportunities to develop the following skills for learning, skills for life and skills for work. Some of these opportunities are described in the table below:

Skills for learning, skills for life and skills for work	Suggested approaches for learning and teaching
Numeracy is the ability to use numbers to solve problems by counting, doing calculations, measuring, and understanding graphs and charts. This is also the ability to understand the results.	Throughout this Unit, learners will have ample opportunities to use number in both real-life and STEM-related practical situations. Learners will have the opportunity to work with information through analysis and interpretation, by drawing conclusions and making deductions and informed decisions.
Applying Applying is the ability to use existing information to solve a problem in a different context, and to plan, organise and complete a task.	Wherever possible, learners should be given the opportunity to apply the skills, knowledge and understanding they have developed to solve mathematical problems in a range of real-life, cross-curricular and STEM-related contexts. Learners could be encouraged to think about how they are going to tackle problems, decide which skills and processes to use and then carry out the processes to complete the task. To determine a learner's level of understanding, learners should be encouraged to show and explain their thinking at all times. At Higher, learners could be encouraged to think creatively to adapt strategies to suit the given problem or situation.
Analysing and evaluating This covers the ability to identify and weigh-up the features of a situation or issue and to use your judgement of them in coming to a conclusion. It includes reviewing and considering any potential solutions.	Wherever possible, learners could be given the opportunity to identify real-life tasks or STEM situations which require the use of mathematics. Learners should be encouraged to analyse the task or situation to decide how it can be addressed and what mathematical skills will need to be applied. Learners should also be provided with opportunities to interpret the results of their calculations and to draw conclusions. Conclusions drawn by the learner could be used to form the basis of a model for making future choices or decisions.

There may also be further opportunities for the development of additional skills for learning, skills for life and skills for work in the delivery of this Unit. These opportunities may vary and are at the discretion of the centre.

Approaches to assessment and gathering evidence

The purpose of this section is to give advice and guidance on approaches to integrating assessment within this Unit.

The *Applications* Unit can be assessed in a variety of ways and could include for example:

- a project or investigation
- problem solving tasks or activities
- short/extended response tests

These approaches are not exhaustive and other possibilities also exist.

The following table gives some examples of how these approaches could be used within the Unit to provide a varied and integrated assessment experience. This approach aims to make the assessment process more coherent and meaningful for learners.

The sequencing and integration of assessment for this Unit could also mirror the examples described in the section on 'Approaches to learning and teaching'.

Approach to assessment	**Examples of approaches to assessment**
Project/ investigation	Mathematical investigations are fundamental to the study of mathematics and often lead to an appreciation of how mathematics can be applied. An investigation could be carried out into the study of centroids, orthocentres and circumcentres.
Problem solving tasks or activities	Learners could be given a discrete task or activity which would provide the opportunity to gather evidence. Learners could be asked to use different methods of finding the area under a curve (trapezoidal, Simpson's, integration).
Short/extended response tests	Learners could be given a short answer or extended response test. Questions could be asked which require the use of recurrence relations.

It would normally be expected that considerable learning and teaching would have taken place prior to the collection of evidence for assessment purposes.

Further advice and guidance on assessment for the Mathematics Course and its components Units is contained within the *Course Support Notes.*

Exemplification of assessment is provided in Unit assessment support.

When delivering this Unit as part of the Higher Mathematics Course, reference should be made to the appropriate content statements within the 'Further mandatory information on Course coverage' section in the *Course Assessment Specification*.

Equality and inclusion

Information about equality and inclusion issues related to this and other Units in Mathematics is given in the Higher Mathematics *Course Support Notes*.

It is recognised that centres have their own duties under equality and other legislation and policy initiatives. The guidance given in these *Unit Support Notes* is designed to sit alongside these duties but is specific to the delivery and assessment of the Unit.

Alternative approaches to Unit assessment to take account of the specific needs of learners can be used. However, the centre must be satisfied that the integrity of the assessment is maintained and that alternative approaches to assessment will, in fact, generate the necessary evidence of achievement.

Appendix 1: Reference documents

The following reference documents will provide useful information and background.

- Assessment Arrangements (for disabled learners and/or those with additional support needs) — various publications on SQA's website: http://www.sqa.org.uk/sqa/14976.html
- *Building the Curriculum 4: Skills for learning, skills for life and skills for work*
- *Building the Curriculum 5: A framework for assessment*
- Course Specifications
- Design Principles for National Courses
- *Guide to Assessment (June 2008)*
- *Principles and practice papers for curriculum areas*
- *Research Report 4 — Less is More: Good Practice in Reducing Assessment Time*
- *Coursework Authenticity — a Guide for Teachers and Lecturers*
- *SCQF Handbook: User Guide* (published 2009) and SCQF level descriptors (reviewed during 2011 to 2012): www.sqa.org.uk/sqa/4595.html
- *SQA Skills Framework: Skills for Learning, Skills for Life and Skills for Work*
- SQA Guidelines on e-assessment for Schools
- SQA Guidelines on Online Assessment for Further Education
- SQA e-assessment web page: www.sqa.org.uk/sqa/5606.html

Administrative information

Published: June 2014 (version 1.0)

History of changes to Unit Support Notes

Version	Description of change	Authorised by	Date